Problems

(1) Use symmetry to evaluate the double integrals.

(a) \[ \int_{\mathcal{R}} \sin x \, dA, \quad \mathcal{R} = [0, 2\pi] \times [0, 2\pi]. \]

(b) \[ \int_{\mathcal{R}} x^3 \, dA, \quad \mathcal{R} = [-4, 4] \times [0, 5]. \]

(2) Evaluate the iterated integrals.

(a) \[ \int_{-1}^{1} \int_{0}^{\pi} x^2 \sin y \, dy \, dx \]

(b) \[ \int_{0}^{1} \int_{0}^{2} (x + 4y^3) \, dx \, dy \]

(3) Evaluate the following integrals using Fubini’s theorem.

(a) \[ \int_{0}^{1} \int_{0}^{1} y \sqrt{1 + xy} \, dy \, dx \]

(b) \[ \int_{0}^{1} \int_{0}^{1} xe^{xy} \, dx \, dy \]

(4) Compute the double integral over the domain \( \mathcal{D} \) indicated

(a) \( f(x, y) = x; \ 0 \leq x \leq 1, \ 1 \leq y \leq e^{x^2}. \)

(b) \( f(x, y) = \sin x; \ \text{bounded by } x = 0, \ x = 1, \ y = \cos x. \)

(5) Find the volume of the region bounded by \( z = 40 - 10y, \ z = 0, \ y = 0, \) and \( y = 4 - x^2. \)

(6) Find the height of the “ceiling” in Figure 30 (page 2) defined by \( z = y^2 \sin x \) for \( 0 \leq x \leq \pi, \ 0 \leq y \leq 1. \)

(7) Find the triple integral of the function \( z \) over the ramp in the picture on page 2. Here, \( z \) is the height above the ground.