Tutorial 26.1. Improper integrals
State whether the following improper integral is convergent or divergent. If it is convergent, what is its value?

\[ \int_{-\infty}^{\infty} e^{-2|x|} \, dx \]

(Recall that when \( x < 0 \), \(|x| = -x\))

Tutorial 26.2. Separation of variables
Solve the following initial value problem:

\[ \frac{dy}{dx} = \frac{y^2}{x}; \quad y(e) = 3 \]

Tutorial 26.3. First order linear equations
Solve the following differential equation subject to the given initial condition:

\[ x \frac{dy}{dx} + (1 + x)y = 3; \quad y(4) = 50 \]

Tutorial 26.4. Predator-Prey
Suppose the system of equations

\[ \begin{align*}
\frac{dy}{dt} &= 4y - xy \\
\frac{dx}{dt} &= -3x + 5xy
\end{align*} \]

defines the influence of the populations (in thousands) of two competing species on their growth rates.
(a) Find an equation relating \( x \) and \( y \) assuming \( y = 1 \) when \( x = 1 \).
(b) Find values of \( x \) and \( y \) such that both populations are constant.

Tutorial 26.5. Equilibria
Find all equilibrium points and determine their stability:

\[ \frac{dy}{dx} = (1 - e^y)(y - 4) \]

Tutorial 26.6. Partial derivatives
For the following function \( f \), determine all points where \( f_x(x, y) = 0 \) and \( f_y(x, y) = 0 \) (ie, find the critical points of \( f \)):

\[ f(x, y) = 9xy - x^3 - y^3 - 6 \]
Tutorial 26.7. Taylor polynomials
Find the Taylor polynomials of degree 2 at 0 and (0,0) respectively of the following functions:
(a) \[ f(x) = e^{-2x+1} \]
(b) \[ f(x, y) = e^2 x \cos(4xy - 1) \]

Tutorial 26.8. Second derivative test
Find the critical points of the following function and determine whether they are saddle points or extrema (or neither):
\[ f(x, y) = x^2 + xy - 2x - 2y + 2 \]