WORD PROBLEMS AND PREDATOR PREY MODELS

Tutorial 23.1. The system of equations
\[ \frac{dy}{dt} = 4y - 2xy \]
\[ \frac{dx}{dt} = -3x + 2xy \]
describes the influence of the populations (in thousands) of two competing species on their growth rates.
(a) Find an equation relating \( x \) and \( y \), assuming \( y = 1 \) when \( x = 1 \).
(b) Find values of the populations so that both populations are constant.

Tutorial 23.2. Salt concentration
Suppose a tank contains 500 liters of a solution of dissolved salt and water (that is kept uniform by stirring). If pure water is allowed to flow into the tank at the rate of 6L/min and the mixture flows out at the rate of 4L/min, how much salt will remain in the tank after 20 minutes if 5kg of salt are in the mixture initially?

Tutorial 23.3. Spread of an epidemic
An influenza epidemic spreads at a rate proportional to the product of the number of people infected and the number not yet infected. Assume that 100 people are infected at the beginning of the epidemic in a community of 20 000 people, and 400 people are infected 10 days later.
(a) Write an equation for the number of people infected, \( y \).
(b) When will half the community be infected?