DIFFERENTIAL EQUATIONS 2

Tutorial 22.1. Linear differential equations
Solve the following differential equations, subject to the given initial conditions:
(a) \[ x \frac{dy}{dx} - 2y = 5x^2, \quad y(1) = 7 \]
(b) \[ x \frac{dy}{dx} + (3 + x)y = 2, \quad y(4) = 10 \]

Tutorial 22.2. The system of equations
\[
\begin{align*}
\frac{dy}{dt} &= 4y - 2xy \\
\frac{dx}{dt} &= -3x + 2xy
\end{align*}
\]
describes the influence of the populations (in thousands) of two competing species on their growth rates.
(a) Find an equation relating \( x \) and \( y \), assuming \( y = 1 \) when \( x = 1 \).
(b) Find values of the populations so that both populations are constant.

Tutorial 22.3. Suppose a person weighing \( w \) pounds has a daily intake of \( C(w) \) calories and uses 17.5\( w \) calories per day. Assume that the person’s change in weight \( dw/dt \) is proportional to the net excess or deficit \( C - 17.5w \) in calories per day.
(a) What happens if \( C = 17.5w \)?
(b) Assume \( C \) is constant (what does this mean?). Write down a differential equation expressing the change in weight. What units must the constant of proportionality have?
(c) Use the fact that 3500 calories is equivalent to 1 lb to rewrite the differential equation.
(d) Solve the differential equation.
(e) Setting the initial weight to be \( w_0 \) rewrite your solution.
(f) Suppose someone initially weighing 180 lbs adopts a diet of 2500 calories a day. Write their weight function.
(g) What does their weight tend to as they continue with this calorie intake for larger and larger amounts of time?
(h) According to the model, how long would it take the person to reach a weight 2 lbs higher than this limiting weight?