IMPROPER INTEGRALS AND DIFFERENTIAL EQUATIONS

Tutorial 20.1. Improper integrals
For each of the following integrals, determine whether the integral converges or diverges, and if it converges find the value of the integral:
(a) \( \int_{0}^{\infty} \frac{dx}{(x + 1)^2} \)
(b) \( \int_{-\infty}^{-1} \frac{2x - 1}{x^2 - x} \, dx \)
(c) \( \int_{-\infty}^{\infty} xe^{-x^2} \, dx \)

Tutorial 20.2. Exponential growth
The population of the world in the year 1650 was about 500 million and in the year 2010 was 6756 million.
(a) Assuming that the population of the world grows exponentially, find the equation for the population \( P(t) \) in millions in the year \( t \).
(b) Use your answer from part (a) to find the population of the world in the year 1.

Tutorial 20.3. Differential equations
Find the general solution for each differential equation and check your solution.
(a) \( y' = -x^3 + 2x^5 \)
(b) \( y' = xy \)