PRELIM 2 REVIEW

Here is a selection of problems to help you prepare for the prelim. Choose which problems will be most worthwhile for YOU to work on, do not just start from the beginning! If there is something you think would be more helpful to work on and ask me about (I didn’t include questions on Riemann sums and trigonometric functions, for example), you’re welcome to do that instead.

**Tutorial 17.1.** Relative extrema and concavity
Find the relative extrema of the following functions and determine where they are increasing/ decreasing and concave up/ concave down.
(a) \( f(x) = x^3 e^x \)
(b) \( f(x) = \frac{\ln x}{x^2} \)

**Tutorial 17.2.** Exercise 91 chapter 5.3
The probability \( P \) that a plant will grow to radius \( R \) can be described by the equation
\[
P(R) = \frac{1}{1 + 2\pi DR^2}
\]
where \( D \) is the density of the plants in an area. A graph of this probability function shows an inflection point around \( R = 0.022 \). Find an expression for the value of \( R \) in terms of \( D \) at the inflection point and find the value of \( D \) corresponding to an inflection point at \( R = 0.022 \).

**Tutorial 17.3.** Absolute extrema
Find the absolute extrema of the following functions (if they exist)
(a) \( f(x) = x \ln x \)
(b) \( f(x) = (x^2 - 4)^{1/3} \)

**Tutorial 17.4.** Graph sketching
Sketch the function
\[
f(x) = \frac{3x^2}{x^2 + 5}
\]
**Tutorial 17.5.** A farmer is constructing a rectangular pen with one additional fence across its width. Find the maximum area that can be enclosed with 2400 m of fencing.

**Tutorial 17.6.** Substitution
Compute the following integrals using substitution:
(a) \[ \int x \sqrt{1 - x} \, dx \]
(b) \[ \int \frac{10\sqrt{y}}{2\sqrt{y}} \, dy \]

**Tutorial 17.7.** Definite integrals
Compute the following definite integrals:
(a) \[ \int_{4}^{9} 4\sqrt{r} - 3r\sqrt{r} \, dr \]
(b) \[ \int_{1}^{3} \frac{\sqrt{\ln x}}{x} \, dx \]

**Tutorial 17.8.** Integration by parts
Compute the following integrals:
(a) \[ \int x^3 \ln x \, dx \]
(b) \[ \int 2t^2 e^{-t} \, dt \]

**Tutorial 17.9.** Solid of revolution
Find the volume of the solid of revolution formed by rotating about the x-axis the area bounded by \( f(x) = 4 - x^2 \) and the x-axis.
Find the average value of the function \( f \) on the interval \([-2, 2]\).