In calculus, we do a lot of adding. We will introduce two “fancy adding machines” in the next couple of days. The first one uses $\sum$ and is called Sigma Notation.

Example:

$$\sum_{n=1}^{5} (2n) = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30$$

**Your turn!** Find the sum of:

$$\sum_{k=3}^{9} (k^2 + 1)$$

**Solution:**

$$\sum_{k=3}^{9} (k^2 + 1) = (3^2 + 1) + (4^2 + 1) + (5^2 + 1) + (6^2 + 1) + (7^2 + 1) + (8^2 + 1) + (9^2 + 1)$$

$$= 10 + 17 + 26 + 37 + 50 + 65 + 82$$

$$= 287$$
We have a few formulæ for sums that show up frequently.

\[
\sum_{k=1}^{n} k = \frac{n(n+1)}{2}
\]

\[
\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}
\]

\[
\sum_{k=1}^{n} k^3 = \left[ \frac{n(n+1)}{2} \right]^2
\]

Use what we know about sums and the above formulæ to evaluate

1. \[\sum_{k=1}^{17} (2 + k) = \]
   SOLUTION:
   \[
   \sum_{k=1}^{17} (2 + k) = \sum_{k=1}^{17} 2 + \sum_{k=1}^{17} k = 17 \cdot 2 + \frac{17(17+1)}{2} = 34 + 153 = 187
   \]

2. \[\sum_{k=18}^{71} k(k - 1) = \]
   SOLUTION:
   \[
   \sum_{k=18}^{71} k(k - 1) = \sum_{k=18}^{71} k^2 - k
   \]
   \[
   = \sum_{k=18}^{71} k^2 - \sum_{k=18}^{71} k
   \]
   \[
   = \left( \sum_{k=1}^{71} k^2 \right) - \left( \sum_{k=1}^{17} k \right) - \left( \sum_{k=1}^{17} k \right)
   \]
   \[
   = \left( \frac{71(71+1)(2 \cdot 71 + 1)}{6} \right) - \frac{17(17+1)(2 \cdot 17 + 1)}{6} - \frac{17(17+1)}{2}
   \]
   \[
   = 117648
   \]

3. \[\sum_{k=1}^{5} \frac{k^3}{225} + \left( \sum_{k=1}^{5} k \right)^3 = \]
   SOLUTION:
   \[
   \sum_{k=1}^{5} \frac{k^3}{225} + \left( \sum_{k=1}^{5} k \right)^3 = \frac{1}{225} \sum_{k=1}^{5} k^3 + \left( \frac{5(5+1)}{2} \right)^3
   \]
   \[
   = \frac{1}{225} \left( \frac{5(5+1)}{2} \right)^2 + \left( \frac{5(5+1)}{2} \right)^3
   \]
   \[
   = \frac{1}{225} \times 15^2 + 15^3 = 1 + 3375 = 3376
   \]