§8.9: Numerical Integration
§9.1: Arc Length and Surface Area
§9.4: Taylor Polynomials

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One-Page Review

1) There are three numerical approximations to \( \int_a^b f(x) \, dx \):

(a) The midpoint rule: \( M_N = \Delta x \left( f(c_1) + \ldots + f(c_N) \right) \), \( c_j = a + \left( j + \frac{1}{2} \right) \Delta x \).

(b) The trapezoid rule: \( T_N = \frac{1}{2} \Delta x \left( y_0 + 2y_1 + 2y_2 + \ldots + 2y_{N-1} + y_N \right) \)

(c) Simpson's rule: \( S_N = \frac{1}{3} \Delta x \left( y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-2} + 4y_{N-1} + y_N \right) \)

2) The arc length of \( f(x) \) on the interval \([a, b] \)

3) The surface area of the surface obtained by rotating the graph of \( f(x) \) around the x-axis for \( a \leq x \leq b \)

4) The \( n \)-th Taylor Polynomial centered at \( x = a \) for the function \( f \) is

\[
T_n(x) = 
\]

5) The error for the \( n \)-th Taylor Polynomial is

\[
|T_n(x) - f(x)| \leq 
\]

where \( K \) is the maximum of \( |f^{(n+1)}(u)| \) over all \( u \) between \( a \) and \( x \).

6) Taylor's Theorem says that

\[
R_n(x) = T_n(x) - f(x) = 
\]
PROBLEMS

(1) Find the $T_4$ approximation for $\int_0^4 \sqrt{x} \, dx$.

(2) State whether $M_{10}$ underestimates or overestimates $\int_1^4 \ln(x) \, dx$.

(3) For the curve $y = \ln(\cos x)$ over the interval $[0, \pi/4]$, set up an integral to calculate:
   (a) the arc length.
   (b) the surface area when rotated around the x-axis.

(4) Approximate the arc length of the curve $y = \sin(x)$ over the interval $[0, \pi/2]$ using the midpoint approximation $M_8$.

(5) Find the Taylor polynomials $T_2(x)$ and $T_3(x)$ for $f(x) = \frac{1}{1+x}$ centered at $a = 1$.

(6) Find $n$ such that $|T_n(1.3) - \sqrt{1.3}| \leq 10^{-6}$, where $T_n$ is the Taylor polynomial for $\sqrt{x}$ at $a = 1$. 