7.1 Exponential Functions

7.2 Inverse Functions

7.3 Logarithms

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One-Page Review

1. \( f(x) = b^x \) is increasing if \( 1 < b \) and decreasing if \( 0 < b < 1 \).

2. The derivative of \( f(x) = b^x \) is \( \frac{d}{dx} b^x = \frac{\log_b(e)}{e} \).

3. \( \frac{\ln(x)}{e} = \frac{\log_b(x)}{e} \) and \( \frac{f(x)}{e} = \frac{\log_b(x)}{e} \) and \( \frac{\log(bx+b)}{e} = \frac{\log_b(x)}{e} \).

4. \( \int e^x \, dx = \int e^{kx+b} \) for constants \( k, b \).

5. A function \( f \) with domain \( D \) is one to one if \( f \) is increasing on \( D \).

6. Let \( f \) have domain \( D \) and range \( R \). The inverse \( f^{-1} \) is the unique function with domain \( R \) and range \( D \) such that \( f(f^{-1}(x)) = x \) for all \( x \) in \( R \).

7. The inverse of \( f \) exists if and only if \( f \) is one to one.

8. Horizontal Line Test: \( f \) is one-to-one if and only if every horizontal line intersects the graph of \( f \) at most once.

9. To find the inverse function, solve \( y = f(x) \) for \( x \) in terms of \( y \).

10. The graph of \( f^{-1} \) is obtained by reflecting the graph of \( f \) through the line \( y = x \).

11. If \( f \) is differentiable and one-to-one with inverse \( g \), then for \( x \) such that \( f'(g(x)) \neq 0 \),

\[
g'(x) = \frac{1}{f'(g(x))}.
\]

12. The inverse of \( f(x) = b^x \) is \( \log_b(x) \).

13. Logarithm Rules
   
   a) \( \log_b(1) = 0 \) and \( \log_b(b) = 1 \).
   
   b) \( \log_b(xy) = \log_b(x) + \log_b(y) \) and \( \log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \).
   
   c) Change of Base: \( \frac{\log_a(x)}{\log_a(b)} = \log_b(x) \).
   
   d) \( \log_b(x^n) = n \log_b(x) \).

14. \( \frac{\ln(x)}{x} = \frac{1}{x} \) and \( \frac{\log_b(x)}{\log_b(x)} = 1 \).

15. \( \int \frac{1}{x} \, dx = \ln|x| \).
PROBLEMS

(1) Calculate the derivative.

(a) \( f(x) = 7e^{2x} + 3e^{4x} \)
(b) \( f(x) = e^{ex} \)
(c) \( f(x) = 3x \)
(d) \( f(t) = \frac{1}{1 - e^{-3t}} \)
(e) \( f(t) = \cos(\text{te}^{-2t}) \)
(f) \( \int e^{x} \sin t \ dt \)
(g) \( f(x) = x \ln x \)
(h) \( f(x) = \ln(x^5) \)
(i) \( f(x) = \ln(\sin(x) + 1) \)
(j) \( f(x) = e^{\ln(x)^2} \)
(k) \( f(x) = \log_a(\log_b(x)) \)
(l) \( f(x) = 16\sin x \)

(2) Calculate the integral.

(a) \( \int \frac{7}{x} \ dx \)
(b) \( \int e^{4x} \ dx \)
(c) \( \int \frac{\ln x}{x} \ dx \)
(d) \( \int \frac{1}{9x-3} \ dx \)
(e) \( \int_2^3 (e^{4t-3}) \ dt \)
(f) \( \int e^t \sqrt{e^t+1} \ dt \)
(g) \( \int e^x \cos e^x \ dx \)
(h) \( \int \tan(4x + 1) \ dx \)

(3) For each function shown below, sketch the graph of the inverse.

(A) \( f(x) = x + \cos x, \ b = 1 \)
(B) \( f(x) = 4x^3 - 2x, \ b = -2 \)
(C) \( f(x) = \sqrt{x^2 + 6x} \text{ for } x \geq 0, \ b = 4 \)
(D) \( f(x) = \frac{1}{x+1}, \ b = \frac{1}{4} \)

(4) Calculate \( g(b) \) and \( g'(b) \), where \( g \) is the inverse of \( f \).

(a) \( f(x) = x + \cos x, \ b = 1 \)
(b) \( f(x) = 4x^3 - 2x, \ b = -2 \)
(c) \( f(x) = \sqrt{x^2 + 6x} \text{ for } x \geq 0, \ b = 4 \)
(d) \( f(x) = \frac{1}{x+1}, \ b = \frac{1}{4} \)

(5) Which of the following statements are true and which are false? If false, modify the statement to make it correct.

(a) If \( f \) is increasing, then \( f^{-1} \) is increasing.
(b) If \( f \) is concave up, then \( f^{-1} \) is concave up.
(c) If \( f \) is odd then \( f^{-1} \) is odd.
(d) Linear functions \( f(x) = ax + b \) are always one-to-one.
(e) \( f(x) = \sin(x) \) is one-to-one.