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**ONE PAGE REVIEW**

(1) The graph of $x = f(y)$ is the graph of $y = f(x)$ reflected across the line ______

(2) The area between $f(x)$ and $g(x)$ from $a$ to $b$ is ______

(3) The ______ of $f(x)$ over the interval $[a, b]$ is ______

(4) The **Mean Value Theorem for Integrals** says that if $f$ is continuous on $[a, b]$ with mean value $M$, then there is some $c \in [a, b]$ such that ______

(5) If a shape has cross-sectional area $A(y)$ and height extends from $y = a$ to $y = b$, then it's volume is ______

(6) **Cavilieri’s Principle** says if two solids have equal ______, then they also have equal ______

(7) **The Disk Method**: If $f(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region under the graph around the x-axis has volume ______

(8) **The Washer Method**: If $f(x) \geq g(x) \geq 0$ on $[a, b]$, then the solid obtained by rotating the region between $f(x)$ and $g(x)$ around the x-axis has volume ______
PROBLEMS

(1) Sketch the region enclosed by the curves and set up an integral to compute it’s area, but do not evaluate.

(a) \( y = 4 - x^2, y = x^2 - 4 \)
(b) \( y = x^2 - 6, y = 6 - x^3, x = 0 \)
(c) \( y = x\sqrt{x - 2}, y = -x\sqrt{x - 2}, x = 4 \)
(d) \( x = 2y, x + 1 = (y - 1)^2 \)
(e) \( y = \cos x, y = \cos(2x), x = 0, x = \frac{2\pi}{3} \)

(2) Calculate the volume of a cylinder inclined at an angle \( \theta = \frac{\pi}{6} \) with height 10 and base of radius 4.

\[ \text{Volume} = \pi \times 4^2 	imes 10 \times \cos \frac{\pi}{6} \]

(3) Calculate the volume of the ramp in the figure below in three ways by integrating the area of the cross sections:

(a) perpendicular to the x-axis.
(b) perpendicular to the y-axis.
(c) perpendicular to the z-axis.

(4) Let \( M \) be the average value of \( f(x) = 2x^2 \) on \([0, 2]\). Find a value \( c \) such that \( f(c) = M \).

\[ M = \frac{\int_0^2 2x^2 \, dx}{2-0} \]

(5) Find the flow rate through a tube of radius 2 meters, if it’s fluid velocity at distance \( r \) meters from the center is \( v(r) = 4 - r^2 \).
(6) Compute the volume of a cone of height 12 whose base is an ellipse with semimajor axis \(a = 6\) and semiminor axis \(b = 4\).

(7) Sketch the region enclosed by the curves, and determine the cross section perpendicular to the \(x\)-axis. Set up an integral for the volume of revolution obtained by rotating the region around the \(x\)-axis, but do not evaluate.

(a) \(y = x^2 + 2, \ y = 10 - x^2\).
(b) \(y = 16 - x, \ y = 3x + 12, \ x = -1\).
(c) \(y = \frac{1}{x}, \ y = \frac{5}{2} - x\).
(d) \(y = \sec x, \ y = 0, \ x = -\frac{\pi}{4}, \ x = \frac{\pi}{4}\).

(8) A frustum of a pyramid is a pyramid with its top cut off. Let \(V\) be the volume of a frustum of height \(h\) whose base is a square of side \(a\) and whose top is a square of side \(b\) with \(a > b > 0\).

(a) Show that if the frustum were continued to a full pyramid (i.e. the top wasn’t cut off), it would have height \(h a / (a - b)\).
(b) Show that the cross sectional area at height \(x\) is a square of side \((1/h)(a(h - x) + bx)\).
(c) Show that \(V = \frac{1}{3}h(a^2 + ab + b^2)\).