(1) The arc length of \( f(x) \) on the interval \([a, b]\) is \( \sqrt{1 + (f'(x))^2} \). 

(2) The surface area of the surface obtained by rotating the graph of \( f(x) \) around the x-axis for \( a \leq x \leq b \) is \( 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, dx \). 

(3) The \( n \)-th Taylor Polynomial centered at \( x = a \) for the function \( f \) is 

\[
T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k
\]

(4) The error for the \( n \)-th Taylor Polynomial is 

\[
|T_n(x) - f(x)| \leq \frac{M}{(n+1)!} (x-a)^{n+1}
\]

(5) Taylor’s Theorem says that 

\[
R_n(x) = T_n(x) - f(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}
\]

(6) A differential equation is like a normal equation, except you solve a differential equation for a function instead of a number. 

(7) The order of a differential equation is the highest derivative of \( y \) appearing in the equation. 

<table>
<thead>
<tr>
<th>Equation</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y' = x^2 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( y''' + x^4 y' = 2 )</td>
<td>( 3 )</td>
</tr>
<tr>
<td>( (y')^3 + yy' = \sin x )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( y'' = y^2 )</td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

(8) The technique for solving a differential equation where you move all the \( x \)-terms to one side and all of the \( y \)-terms to the other side is called separable differential equations.
Practice Problems

§9.1 (Arc Length and Surface Area)  §9.4 (Taylor Polynomials)  §10.1 (Differential Equations)

(1) For the curve curve $y = \ln(\cos x)$ over the interval $[0, \pi/4]$, set up an integral to calculate:
   (a) the arc length.
   (b) the surface area when rotated around the x-axis.

(2) Approximate the arc length of the curve $y = \sin(x)$ over the interval $[0, \pi/2]$ using the
midpoint rule $M_8$.

(3) Find the Taylor polynomials $T_2(x)$ and $T_3(x)$ for $f(x) = \frac{1}{1+x}$ centered at $a = 1$.

(4) Find $n$ such that $|T_n(1.3) - \sqrt{1.3}| \leq 10^{-6}$, where $T_n$ is the Taylor polynomial for $\sqrt{x}$ at $a = 1$.

(5) Find the general solutions of the following differential equations using separation of variables.
   (a) $\frac{dy}{dt} - 2y = 1$
   (b) $(1 + x^2)y' = x^3y$

(6) Solve the initial value problem
\[
\begin{align*}
y' + 2y &= 0 \\
y(\ln(2)) &= 3
\end{align*}
\]