§7.1 (Exponential Functions), §7.2 (Inverse functions), §7.3 (Logarithms) September 29, 2016

(1) \( f(x) = b^x \) is increasing if \( b > 1 \) and decreasing if \( 0 < b < 1 \).

(2) The derivative of \( f(x) = b^x \) is \( \frac{d}{dx} b^x = \frac{d}{dx} e^{\ln b x} = \) \( e^{\ln b x} \cdot \ln b = b^x \ln b \).

(3) \( \frac{d}{dx} e^x = e^x \) and \( \frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x) \) and \( \frac{d}{dx} e^{kx+b} = e^{kx+b} \cdot k = e^{kx+b} \cdot k \).

(4) \( \int e^x \, dx = e^x \) and \( \int e^{kx+b} \, dx = \frac{1}{k} e^{kx+b} \) for constants \( k, b \).

(5) A function \( f \) with domain \( D \) is one to one if \( f \) is increasing or decreasing on its domain.

(6) Let \( f \) have domain \( D \) and range \( R \). The inverse \( f^{-1} \) is the unique function with domain \( R \) and range \( D \) such that \( f(f^{-1}(y)) = y \) for every \( y \) in \( R \).

(7) The inverse of \( f \) exists if and only if \( f \) is one to one.

(8) Horizontal Line Test: \( f \) is one-to-one if and only if every horizontal line intersects the graph of \( f \) at most once.

(9) To find the inverse function, solve \( y = f(x) \) for \( x \) in terms of \( y \).

(10) The graph of \( f^{-1} \) is obtained by reflecting the graph of \( f \) through the line \( y = x \).

(11) If \( f \) is differentiable and one-to-one with inverse \( g \), then for \( x \) such that \( f'(g(x)) \neq 0 \),

\[
g'(x) = \frac{1}{f'(g(x))}.
\]

(12) The inverse of \( f(x) = b^x \) is \( \log_b(x) \).

(13) Logarithm Rules

\( \text{(a) } \log_b(1) = 0 \) and \( \log_b(b) = 1 \).

\( \text{(b) } \log_b(xy) = \log_b(x) + \log_b(y) \) and \( \log_b \left( \frac{x}{y} \right) = \log_b(x) - \log_b(y) \).

\( \text{(c) Change of Base: } \frac{\log_b(x)}{\log_b(a)} = \log_a(x) \).

\( \text{(d) } \log_b(x^n) = n \log_b(x) \).

(14) \( \frac{d}{dx} \ln(x) = \frac{1}{x} \) and \( \frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)} \).

(15) \( \int \frac{1}{x} \, dx = \ln(x) \).
(1) Calculate the derivative.
   (a) \( f(x) = 7e^{2x} + 3e^{4x} \)
   (b) \( f(x) = e^{x^2} \)
   (c) \( f(x) = 3^x \)
   (d) \( f(t) = \frac{1}{1 - e^{-3t}} \)
   (e) \( f(t) = \cos(te^{-2t}) \)
   (f) \( \int e^x \sin t \, dt \)
   (g) \( f(x) = x \ln x \)
   (h) \( f(x) = \ln(x^5) \)
   (i) \( f(x) = \ln(\sin(x) + 1) \)
   (j) \( f(x) = e^{\ln(x)^2} \)
   (k) \( f(x) = \log_a(\log_b(x)) \)
   (l) \( f(x) = 16^{\ln x} \)

(2) Calculate the integral.
   (a) \( \int (e^x + 2) \, dx \)
   (b) \( \int \frac{7}{x} \, dx \)
   (c) \( \int e^{4x} \, dx \)
   (d) \( \int \frac{\ln x}{x} \, dx \)
   (e) \( \int \frac{1}{9x - 3} \, dx \)
   (f) \( \int_2^3 (e^{t-3}) \, dt \)
   (g) \( \int e^t \sqrt{e^t + 1} \, dt \)
   (h) \( \int e^x \cos e^x \, dx \)
   (i) \( \int \tan(4x + 1) \, dx \)
   (j) \( \int x^3 \, dx \)
(3) For each function shown below, sketch the graph of the inverse.

(4) Calculate \( g(b) \) and \( g'(b) \), where \( g \) is the inverse of \( f \).
   
   (a) \( f(x) = x + \cos x, \ b = 1 \).
   
   (b) \( f(x) = 4x^3 - 2x, \ b = -2 \).
   
   (c) \( f(x) = \sqrt{x^2 + 6x} \) for \( x \geq 0, \ b = 4 \).
   
   (d) \( f(x) = \frac{1}{x+1}, \ b = \frac{1}{4} \).

(5) Which of the following statements are true and which are false? If false, modify the statement to make it correct.

   (a) If \( f \) is increasing, then \( f^{-1} \) is increasing.
   
   (b) If \( f \) is concave up, then \( f^{-1} \) is concave up.
   
   (c) If \( f \) is odd then \( f^{-1} \) is odd.
   
   (d) Linear functions \( f(x) = ax + b \) are always one-to-one.
   
   (e) \( f(x) = \sin(x) \) is one-to-one.