Test 2  
Math 481  
Tuesday April 5 in class

Work out the complete tableaux (if finite, otherwise a finished tableau) in each case and then work the rest of each problem.

1. \( F((A \land B) \implies \neg A) \) Specify a non-contradictory branch, give a truth assignment to \( A, B \) which makes \((A \lor B) \implies (B \lor C)\) false and agrees with the branch.

\[
\begin{align*}
F(A \land B) & \rightarrow \neg A \\
T(A \land B) & \\
F \neg A & \\
T A & \\
T A & \\
T B &
\end{align*}
\]

This tableau is finished and its one branch is non-contradictory. A truth assignment agreeing with this branch which makes the sentence false is \( T A, T B \).

2. \( F((A \lor (B \land C)) \implies ((A \lor B) \land (A \lor C))) \)

Show that that every branch has a contradiction and conclude that \((A \lor (B \land C)) \implies ((A \lor B) \land (A \lor C)))\) is tableaux provable.
Since all four branches lead to a contradiction, any valuation will make the sentence true.

3. $F( (\forall x)(\forall y)A(x, y) ) \rightarrow ( (\forall y)(\exists x)A(x, y))$ 

Show that every branch has a contradiction and conclude that $F( (\forall x)(\forall y)A(x, y) ) \rightarrow ( (\forall y)(\exists x)A(x, y))$ is tableaux provable.

Since there is only one branch and it is contradictory, any model interpreting the sentence must satisfy it, i.e. the sentence is tableaux provable.
4) \( F((\exists x)(A(x) \land B(x))) \implies (\forall x)A(x) \)

Use a non-contradictory branch to exhibit a structure with domain the constants on that branch which makes \( F((\exists x)(A(x) \land B(x))) \implies (\forall x)A(x) \) false. 

\[
\begin{align*}
T(\exists x)(A(x) \land B(x)) & \\
F(\forall x)A(x) & \\
T(A(a) \land B(a)) & \\
TA(a) & \\
TB(a) & \\
FA(b) & \\
\end{align*}
\]

A structure which makes the sentence false is: Domain = \{a, b\}, \( A = \{a\} \), \( B = \{a, b\} \).

5) Give a statement ? in statement logic based on \( \land, \lor, \neg \), with the following truth table.

<table>
<thead>
<tr>
<th>p</th>
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Reading off the truth table we get that a candidate for ? is \( (p \land q \land r) \lor (p \land q \land \neg r) \lor (\neg p \land \neg q \land \neg r) \).