The purpose of this Homework is to finish the examples of special relativity seen during class.

We will consider a universe with a single spatial dimension. We will describe our events using coordinates \((t, x)\), where \(t\) is the time, and \(x\) is the position. Furthermore, we will normalize our coordinates in such a way that 
\[
c = \text{speed of light} = 1.
\]

Let \(r_1\) and \(r_2\) denote the position of two particles, whose graph in \(\mathbb{R}^2\) are given parametrically by the equations
\[
r_1(t) = (t, 0) \quad \text{and} \quad r_2(t) = ((\cosh 1)t, (\sinh 1)t).
\]

1. Compute the relative velocity between \(r_1\) and \(r_2\).

2. Use the properties of \(\cosh\) and \(\sinh\) to check that if 
\[
g(-1) = \begin{bmatrix}
\cosh (-1) & \sinh (-1) \\
\sinh (-1) & \cosh (-1)
\end{bmatrix},
\]
then
\[
g(-1)r_1(t) = ((\cosh 1)t, -(\sinh 1)t) \quad \text{and} \quad g(-1)r_2(t) = (t, 0).
\]

3. We can use the parametric equation 
\[
a(t) = (t, t),
\]
to describe one the light rays coming out of the origin. Convince yourself that the relative velocity of this light ray with respect to \(r_1\) and \(r_2\) is 1. (Hint: use the change of coordinates described in problem 2)

4. Let 
\[
p_1 = (1, 0.2) \quad \text{and} \quad p_2 = (1, -0.2)
\]
be two events that occur at the same time for observer \(r_1\). Using the change of reference frame described in problem 2, compute the times at which this events occurred according to observer \(r_2\).

5. Explain how problem 4 is related with the example discussed at the lecture. (About the person traveling in the middle of a train and another person watching everything from a platform.)