1. Encrypt the plaintext BOMB using the affine cipher \((9x + 2) \mod 26\).

2. The ciphertext UCR was encrypted using the affine cipher \((9x + 2) \mod 26\). Find the plaintext.

3. The following ciphertext was encrypted by an affine cipher \(\mod 26\): CRWWZ. The plaintext starts: HA. Decrypt the message.

4. Divide \(2^{10203}\) by 101. What is the remainder?

5. Show that \(2^{56} + 3^{56}\) is divisible by 17.

6. (a) Suppose that you know \(m = pq = 11021\), and that you find that \(p + q = 210\). Find \(p\) and \(q\).

(b) Now suppose that \(pq = M\), and \(p + q = N\). Find \(p\) and \(q\) in terms of \(M, N\).

7. Suppose that Alice and Bob have created their own RSA keys: Alice has public key \((e_A, m_A)\) and private key \(d_A\), while Bob’s keys are \((e_B, m_B)\) and \(d_B\). Explain how Alice can send Bob a message that only Bob can read, yet that Bob can be sure that Alice was the one who sent it to her (i.e. how can Alice digitally sign her message?).