You have 1 hour 30 minutes to complete this exam. The exam starts at 7:30pm. Each question is worth 20 marks. There are 5 questions in total. You are free to use results from the lectures, but you should clearly state any theorems you use. The exam is printed on both sides of the paper. Good luck!

(1) (a) Find all the critical points of

\[ f(x, y) = x^2 - 6xy + 10y^2 \]

and determine their nature.

(b) Find the second order Taylor polynomial of

\[ g(x, y) = x^2 e^x - 6x(e^y - 1) - 20(\cos(y) - 1) \]

about the point (0, 0) (you may omit the remainder term).

(2) Calculate

\[ \int_0^1 \int_{\frac{3}{\sqrt{7}}}^1 e^{x^4} \, dx \, dy. \]

(3) (a) The number of cookies which can be baked from \( x \) pounds of flour, \( y \) pounds of sugar and \( z \) pounds of butter is

\[ C(x, y, z) = xyz + 3. \]

Assuming that we can buy a total of six pounds of ingredients, what is the maximum number of cookies which can be produced?

(b) Find the maximum and minimum values of the function \( f(x, y) = x^4 + y^4 \) on the disc \( x^2 + y^2 \leq 2 \).
(4) Suppose \(x, y, z, w\) are related by the following equations.

\[
\begin{align*}
xe^z + 2w + 5y &= 0 \\
w + z + yz^5 + 4w^6 &= 0
\end{align*}
\]

(a) Show that these equations uniquely determine \(z\) and \(w\) as functions of \(x\) and \(y\) near the point \((0,0,0,0)\).

(b) Calculate \(\frac{\partial z}{\partial x}|_{(0,0)}\) and \(\frac{\partial w}{\partial x}|_{(0,0)}\).

(5) A paperweight \(P\) consists of the points in \(\mathbb{R}^3\) lying above the triangle with vertices \((0,0,0), (1,0,0)\) and \((0,1,0)\) and below the plane \(z = 3x + 2y\).

(a) Calculate the volume of the paperweight.

(b) Suppose the paperweight is made of a material whose density at the point \((x, y, z)\) is \(f(x, y, z) = 6 - z\). Set up an iterated integral for the mass

\[
M = \iiint_P f(x, y, z) dV
\]

of the paperweight. Be sure to include all limits of integration, but do not attempt to evaluate the integral.