1. (5 pts each) Define each of the following groups by explicitly describing the set and the operation, and answer the questions without proof.
   (i) Define the symmetric group \( S_n \). What is the minimum number of elements of \( S_n \) we need to generate \( S_n \)?
   (ii) Define the symmetry group \( 
\Sigma(\pi_n) \) of a regular \( n \)-gon \( \pi_n \). What is the order of \( \Sigma(\pi_n) \)?

2. (2 pts each) Determine whether each of the following statements is true or false. (Do not give a proof or a counter-example.)
   (i) The group \( \Sigma(\pi_n) \) is isomorphic to the group \( S_n \).
   (ii) The permutation \( (1234) \) is in \( A_4 \).
   (iii) The group \( SL_n(\mathbb{R}) \) is a normal subgroup of \( GL_n(\mathbb{R}) \).
   (iv) The group \( S_n \) is commutative.
   (v) Every permutation is a product of transpositions in a unique way.

3. (20 pts) Let \( H, K \) be normal subgroups of a finite group \( G \).
   (i) Show that \( HK = KH \). Show also that \( HK \) is a subgroup of \( G \).
   (ii) Show that if \( H \cap K = \{1\} \), then \( HK \cong H \times K \).

4. (20 pts) Find all solutions to the following simultaneous congruences.
   \[
   \begin{align*}
   3x & \equiv 2 \pmod{5} \\
   2x & \equiv 1 \pmod{3}
   \end{align*}
   \]

5. (20 pts) Let \( n = p_1^{e_1}p_2^{e_2} \) be the prime decomposition of \( n \in \mathbb{N} \). Show that the Euler function \( \varphi \) satisfies
   \[
   \varphi(n) = n \left( 1 - \frac{1}{p_1} \right) \left( 1 - \frac{1}{p_2} \right).
   \]
   (Hint: show that the group \( U(\mathbb{Z}/n\mathbb{Z}) \) is isomorphic to a direct product of some groups.)

6. (20 pts) For any \( n \geq 3 \), show that \( A_n \) contains a subgroup isomorphic to \( S_{n-2} \).