You are NOT allowed calculators or the text. With the exception of True/False questions, JUSTIFY ALL ANSWERS, SHOW ALL WORK!

1) (a) (10 points) Find a basis for the subspace of all vectors in $\mathbb{R}^4$ orthogonal to both $(1, 0, 2, 3)$ and $(1, 0, 0, -1)$.
(b) (8 points) Let $W \subseteq \mathbb{R}^4$ be the subspace spanned by $\{(0, 2, 0, 7), (1, 1, 1, 4), (-1, 1, -1, 3), (3, 1, 3, 5)\}$. What is the dimension of $W$? Find a matrix $A$ whose left null space is exactly $W$.

2) (8 points) a) Find the matrix $P$ that projects vectors $\vec{v} \in \mathbb{R}^3$ onto the plane $x + 2y - z = 0$.
b) (8 points) Express the vector $(-2, -2, 1)$ as the sum of a vector in the plane from part (a), and a vector normal to that plane.

3) For the questions below, just write the complete word 'TRUE' or 'FALSE' or leave it blank. No explanations are needed for this question only. A correct answer is worth 3 points, leaving a question blank is worth 0 points, and an incorrect answer is worth $-3$ points.
(a) (3/0/−3 points) For any $2 \times 3$ matrix $A$, the null space of $A$ is perpendicular to the null space of $A^T$.
(b) (3/0/−3 points) Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$ be non-zero vectors in $\mathbb{R}^3$. If every pair of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ are perpendicular, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis of $\mathbb{R}^3$.
(c) (3/0/−3 points) Every (positive dimensional) subspace $S$ of $\mathbb{R}^n$ has an orthonormal basis.
(d) (3/0/−3 points) If $\vec{u}$ and $\vec{v}$ are $3 \times 1$, then $\vec{u}\vec{v}^T$ has determinant 0.

4) (16 points) Find the least squares plane of best fit (in $\mathbb{R}^3$) for the four points $(1, 0, 0), (0, 1, 0), (1, 1, 3), (1, 2, 6)$ and that passes through the origin.

5) (a) (7 points) Find the determinant of the matrix
$$
\begin{bmatrix}
1 & 0 & 1 & 0 \\
7 & 1 & 4 & 1 \\
1 & 0 & 7 & 0 \\
0 & 3 & 6 & 2
\end{bmatrix}.
$$
(b) (8 points) Assume that $A$, $B$, and $C$ are invertible $3 \times 3$ matrices, $\det A = a$, $\det B = b$, and $\det C = c$ where $c \neq 0$. Find, with explanation, $\det(A^3)$, $\det(5A)$, $\det(C^T)$ and $\det(A^TB^2C^{-1})$.
(c) (6 points) In terms of $x$ and $a$, find the determinant of
$$
\begin{bmatrix}
   x + a & x + 2a & x + 3a \\
   x + 2a & x + 3a & x + 4a \\
   x + 4a & x + 5a & x + 6a
\end{bmatrix}.
$$

6) (a) (9 points) If $P$ is a projection matrix that projects vectors $\vec{v} \in \mathbb{R}^n$ onto a subspace $S \subseteq \mathbb{R}^n$, prove that $(I - P)^2 = I - P$, where $I$ is the $n \times n$ identity matrix.
b) (8 points) Give an example of a $3 \times 3$ matrix $A$ whose columns are orthonormal and $A$ has exactly one entry equal to 0.