It’s very important to have a feeling for how sequences compare with each other. You will examine six sequences, all of which diverge to $\infty$ as $n \to \infty$. You will determine which of these sequences grow quickly and which grow slowly.

We need a way to compare sequences. Let’s write:

$\{a_n\} \gg \{b_n\}$ to mean that $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$,  

and $\{a_n\} \ll \{b_n\}$ to mean that $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$.

Think of $\gg$ as “infinitely bigger in the limit” and $\ll$ as “infinitely smaller in the limit”.

**Problem:** Arrange the following sequences, deciding which is “first”, “second”, etc., so that:

first $\ll$ second $\ll$ third $\ll$ fourth $\ll$ fifth $\ll$ sixth

(a) $\sqrt{n}$  
(b) $e^n$  
(c) $n!$  
(d) $n^2$  
(e) $\ln n$  
(f) $n^n$