Consider the 2-dimensional random walk defined by \((X_0, Y_0) = (0, 0)\) and \((X_{n+1}, Y_{n+1}) = (X_n, Y_n) + \xi_n\) where \(\xi_1, \xi_2, \ldots\) are independent with

\[
P(\xi_n = (1, 0)) = P(\xi_n = (-1, 0)) = P(\xi_n = (0, 1)) = P(\xi_n = (0, -1)) = \frac{1}{4}.
\]

1. Find a number \(a\) such that \(M_n = X_n^2 + Y_n^2 - an\) is a martingale.

We’d like to estimate the expected time \(ET\) where \(T\) is the time it takes to exit a disk of radius \(r\). In other words, \(T = \min\{n \geq 0 : X_n^2 + Y_n^2 > r^2\}\).

2 Estimate \(E(X_T^2 + Y_T^2)\) when \(r = 10\). (Give upper and lower bounds.)

3 Use parts 1 and 2 to estimate \(ET\).

4 How does your estimate generalize to (a) general radius \(r\)? (b) higher dimensions?