PRELIM. Thursday, March 16, in class, closed book
Covers all material through Week 7.

Reading. 5.1, 5.2


EP7-1. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function such that for all rational numbers $x, y$.

$$f(x + y) = f(x) + f(y).$$

Show that for all real $x$

$$f(x) = x f(1).$$

pp. 152–153: 1,3,5,6; pp. 164–165: 1,2,3,5,7 13

REVIEW FOR PRELIM

Definitions. Countable, uncountable, rational number, field, ordered field, Archimedean property, Cauchy sequence of rationals, equivalence of Cauchy sequences, limit of a sequence, limit-point of sequence, limit-point of set, real number, completeness, open set, closed set, compact set, supremum, upper bound, least upper bound, maximum, lower bound, infimum, greatest lower bound, minimum, limsup, liminf, limit of a function, one-sided limits, dense, interior, closure, continuity, uniform continuity, differentiable, continuously differentiable, affine function.

Results. Countability of the rationals, uncountability of the real numbers, Construction of real numbers from the rationals, bounded monotone sequences have limits, equivalence of definitions of continuity ($\varepsilon - \delta$ and inverse image of open sets), three equivalent ways of determining compactness for subsets of reals (closed and bounded, every sequence has a convergent subsequence, every open cover has a finite subcover), proof that every open cover of $[0, 1]$ has a finite subcover (Heine-Borel theorem), Intermediate Value Theorem, continuous functions take maximum on compact sets, image of compact set under continuous function is continuous, continuous functions on compact sets are uniformly continuous, derivative intermediate value theorem, mean value theorem.