Math 122, Fall 2002
Problem Set 1
Due: Tuesday, September 3

1. Let log denote the base-10 logarithm. What is

\[ \lim_{x \to \infty} \log \log x \] ?

Suppose that \( \log \log x = 4 \). Let \( n \) be the largest integer less than \( x \). How many digits does \( n \) have? Which is larger: \( n \) or the number of atoms in the universe?

2. Let \( n \) be the largest integer less than \( e^{100,000} \). How many digits does \( n \) have? You may use the fact that \( 2.302585 < \ln 10 < 2.302586 \) (you can check this fact on your calculator).

3. Let

\[ f(x) = \int_0^x 1000t^{999} \, dt. \]

What is \( f'(1) \)?

4. Let

\[ f(x) = 67x^6 - 59x^5 + 982x^4 + 6x + 1. \]

Explain (without solving the equation!) why there exist real numbers \( s_1 < 0 < s_2 \) with \( f(s_1) = f(s_2) = 10 \).

5. Let \( f(x) \) be the function whose domain is the set of all real numbers, with \( f(0) = 0 \) and for \( x \neq 0 \),

\[ f(x) = \sin \left( \frac{1}{x} \right). \]

Is \( f \) continuous at \( x = 0 \)? Is it differentiable at \( x = 0 \)? If so, what is \( f'(0) \)?

6. Do the same problem as 5 with except that for \( x \neq 0 \),

\[ f(x) = x \sin \left( \frac{1}{x} \right). \]

7. Do the same problem as 5 with except that for \( x \neq 0 \),

\[ f(x) = x^2 \sin \left( \frac{1}{x} \right). \]