Problem 1. Find the indicated derivatives.
1.a) (5 points) Let \( y(x) = x^5 + 2.5x + \pi^2 \). Find \( y'(x) \).

1.b) (6 points.) \( \frac{d}{d\theta} \sin(\cos(\theta)) \)

1.c) (10 points.) Let \( f(x) = 2\sin(2x) + e^{-x} + x \). Find \( f^{(n)}(x) \), for \( n = 1, 2, 3, 4 \) and 5. (Recall \( f^{(n)} \) denotes the \( n^{th} \) derivative of \( f \).)
Problem 2. Consider the function

\[ f(x) = \begin{cases} 
\sin(x) & \text{if } x < 0 \\
ax + b & \text{if } 0 \leq x < 1 \\
\frac{1}{2}x^2 + \frac{1}{2} & \text{if } 1 \leq x, 
\end{cases} \]

where \( a \) and \( b \) are real numbers.

2.a) \((8 \text{ points})\) For what value of \( a \) and what value of \( b \) is \( f \) continuous at every point in its domain?

2.b) \((8 \text{ points})\) For the values of \( a \) and \( b \) found in 2.a), at which values of \( x \) is \( f \) differentiable?

2.c) \((8 \text{ points})\) For the values of \( a \) and \( b \) found in 2.a), write an expression for \( f'(x) \) on the domain found in part 2.b)
Problem 3. (9 points) Let \( g(x) = 2x^3 + 3x^2 - 12x + 1 \). Find all points \((x, g(x))\) at which the tangent to \( g \) is horizontal. Write equations for all such tangent lines.

Problem 4
4.a) (10 points) Let \( f(x) = \sqrt{x} \), with domain \([0, \infty)\). Use the definition of the derivative to compute \( f'(x) \).

4.b) (4 points) Are there any points in the domain of \( f \) which are not in the domain of \( f' \)? If so, which points?
Problem 5. \textit{(10 points)}

The figure above shows the graph of a function $f$; you do not know the equation for $f$. You do know the values of $f(x_0)$ and $f'(x_0)$. For some small $h$, the point $(x_0 + h, y)$ is on the line $L$, the tangent to the graph of $f$ at $x_0$. Find an expression for $y$.

Problem 6. \textit{(10 points)} Let $f(x) = 2x^2 + x$. Let $x_0 = -1$ and let $\epsilon = \frac{1}{4}$. Find a real number $\delta > 0$ such that

\[ |x - x_0| < \delta \text{ implies } |f(x) - f(x_0)| < \epsilon. \]

Show that your $\delta$ works.
Problem 7 (12 points)

As in the figure above, you are sitting in a tree and swinging a flashlight in the counterclockwise direction. The height of the end of the flashlight (which does not move as you swing) is 1 meter, and you swing the flashlight at an angular rate of $2\pi$ radians per second. Let $\theta$ be the angle between the downwards direction and the direction your flashlight is pointing. When $\theta = \pi/4$, how fast is the beam traveling over the ground?