Exercise 9, which is intended “for people who learned Wedderburn theory in some previous course”, asks you to describe Spec $A$ if $A$ is artinian. What I had in mind was the following. If $A$ is a (possibly non-commutative) artinian ring, then its Jacobson radical $J$ is nilpotent, and $A/J$ is semisimple. By Wedderburn’s theorem $A/J$ is isomorphic to a finite product of matrix rings over division rings.

If we now specialize to the commutative case, it follows that $J$ coincides with the nilradical of $A$ and that $A/J$ is a finite product of fields. Hence every prime ideal is maximal, there are only finitely many such ideals, and Spec $A$ is a finite set with the discrete topology. [It then follows further, from Exercise 3 for instance, that $A$ is a finite product of local artinian rings.]

We may need to use these basic properties of commutative artinian rings at some point. If you don’t know the Wedderburn theory and don’t want to take the time to learn it now, there are self-contained proofs in AM on p. 89.

Incidentally, the nilpotence of the Jacobson radical can be proved very simply if one assumes that $A$ is also noetherian. [This is in fact always the case, and in many examples that arise in practice it is obvious. For example, it is obvious if $A$ is a finite-dimensional algebra over a field.] Under this assumption, consider the descending chain of ideals given by the powers of $J$. This must stabilize to an ideal $I$, which satisfies $JI = I$. Nakayama’s lemma now implies $I = 0$, i.e., $J^n = 0$ for large $n$. [Where did we use the assumption that $A$ is noetherian?]