P1 Let \((a_1, \ldots, a_n)\) be a parking function and let \(0 \leq b_i \leq a_i\) for \(i \in [n]\). Prove that \((b_1, \ldots, b_n)\) is also a parking function.

P2 If \(w = a_1 \ldots a_n \in S_n\), then let \(w^r = a_n \ldots a_1 \in S_n\). Express \(maj(w^r)\) in terms of \(maj(w)\) and \(des(w)\).

P3 Express in terms of the Fibonacci numbers \((F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}, n \geq 3)\) the number of compositions of \(n\) into parts equal to 1 or 2.

P4 Give a combinatorial proof of \(\sum_{k \text{ odd}} \binom{n}{k} = \sum_{k \text{ even}} \binom{n}{k}\).

P5 In how many ways can we pair up \(2n\) people?