Rules: The only person you can communicate with about any of the problems is the instructor, who you are free to consult for clarification of the meaning of any of the problems.

1. In this problem let us call a rectangle whose length equals twice its width a *domino*. 
(a) Show that a $29 \times 70$ rectangle can be covered with a finite number of nonoverlapping dominoes of various sizes, with no two dominoes having the same size.

(b) Find an infinite sequence of integer pairs $(a, b)$ with the g.c.d. of $a$ and $b$ being 1, such that an $a \times b$ rectangle can be covered by finitely many nonoverlapping dominoes all of different sizes.

2. Find all the numbers less than 40 that occur as differences between $a$ and $b$ in primitive Pythagorean triples $(a, b, c)$. How many different primitive triples are there for each difference $a - b$ or $b - a$? (Give a reason for your answer.)

3. (a) Find all integer solutions of the equation $x^2 - xy + y^2 = 49$.
(b) Find formulas for all rational points on the ellipse $x^2 - xy + y^2 = 1$.
(c) Find formulas for all integer solutions of the equation $a^2 - ab + b^2 = c^2$.

4. Find all integer solutions of $85x + 271y = 1$.

5. Using the Farey diagram, find the value of the periodic continued fraction
\[ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \]

6. (a) Compute one period of the periodic separator line in the topograph for the form $Q(x, y) = x^2 - 29y^2$.
(b) Find the continued fraction for \( \sqrt{29} \).
(c) Find the smallest positive integer solutions for the equations $x^2 - 29y^2 = 1$ and $x^2 - 29y^2 = -1$, if these equations do in fact have such solutions.

7. (a) Use a quadratic form to find the continued fraction expansions of \((9 + \sqrt{3})/26\) and \((9 - \sqrt{3})/26\).
(b) Terminology: two quadratic irrational numbers $\alpha$ and $\beta$ are called *conjugates* of each other if they are both roots of the same quadratic equation with rational (or integer) coefficients. For the quadratic irrational $\alpha = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$ find the continued fraction for its conjugate $\beta$ without actually computing the value of $\alpha$ (which would be rather complicated to do).

8. Find the linear fractional transformation that interchanges the two ends of the edge $\langle a/b, c/d \rangle$ of the Farey diagram and preserves orientation of the diagram.