NAME:

QUIZ 4 - MATH 111, Section 19

1. (2 pts each) Find the following derivatives:

(a) \[
\frac{d}{dx} \left( \frac{e^{\sin x}}{x^2 + \pi} \right) = \frac{(x^2 + \pi)e^{\sin x}\cos x - (2x)e^{\sin x}}{(x^2 + \pi)^2}.
\]

(b) \[
\frac{d}{dx}(x^22^x) = (2x)2^x + x^2(2^x \ln 2).
\]

(c) \[
\frac{d}{dx}(\sin t^2 + \cos t^2)
\]

Answer 1: If we think of \( t \) not depending on \( x \), the derivative is zero, because we take derivative with respect to \( x \), not with respect to \( t \).

Answer 2: However, if we see \( t \) as a function of \( x \), then the correct derivative would be

\[
(2t) \cos(t^2)\frac{dt}{dx} - (2t) \sin(t^2)\frac{dt}{dx}
\]

2. (2 pts) Compute \( \lim_{x \to 1} \frac{x^{999} - 1}{x - 1} \).

Answer 1: By the definition of derivative, this limit is exactly \( f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \), for the function \( f(x) = x^{999} \). Then \( f'(x) = 999x^{998} \), hence \( f'(1) = 999 \).

Answer 2: Another way to find this limit is by observing

\[
\lim_{x \to 1} \frac{x^{999} - 1}{x - 1} = \lim_{x \to 1} \frac{(x - 1)(x^{998} + x^{997} + \cdots + 1)}{x - 1} = \lim_{x \to 1} (x^{998} + x^{997} + \cdots + 1) = 999,
\]

where the last equality follows from the fact that the polynomial we get has 999 terms and as we plug in 1 (Direct Substitution), each of the terms equals 1.

3. (2pts) Find the equation of the tangent to the curve \( y = x \sin x \) at the point \((\pi,0)\).

Answer: First, \( \frac{dy}{dx} = \sin x + x \cos x \), so the slope at \( x = \pi \) equals \( \sin \pi + \pi \cos \pi = -\pi \). The equation of the tangent at the given point is then

\[
y - 0 = -\pi(x - \pi),
\]

in other words, \( y = -\pi x + \pi^2 \).