1. (6 pts) Find the point on the line \( y = 4x + 5 \) that is closest to the origin.

**Solution:** Recall that the formula for the distance from the point \((x, y)\) to the origin is \( z = \sqrt{x^2 + y^2} \). So, in our case, \( z = \sqrt{x^2 + (4x + 5)^2} = \sqrt{17x^2 + 40x + 25} \). Then \( z' = \frac{34x + 40}{2\sqrt{17x^2 + 40x + 25}} \). The only critical point for \( z \) is at \( x = \frac{-20}{17} \), i.e. where \( z' = 0 \). Observe that the shortest distance to the origin must be for some \( x \) in between the y-axis intercept and the x-axis intercept, that is, for some \( x \in [-\frac{5}{4}, 0] \). Let us apply the Closed Interval Method to find the absolute minimum:

- \( z \left( -\frac{5}{4} \right) = \sqrt{17 \cdot \frac{25}{16} - 40 \cdot \frac{5}{4} + 25} = \frac{5}{4} \)
- \( z \left( -\frac{20}{17} \right) = \sqrt{17 \cdot \frac{400}{289} - 40 \cdot \frac{20}{17} + 25} = \frac{5\sqrt{17}}{17} \)
- \( z(0) = \sqrt{17 \cdot 0 + 40 \cdot 0 + 25} = 5 \)

Thus, the absolute minimum is achieved at \( x = -\frac{20}{17}, y = \frac{5}{17} \).

2. (2 points) Prove Rolle’s Theorem: Suppose \( y = f(x) \) is differentiable on \([a, b]\). If \( f(a) = f(b) = 0 \), then there is at least one number \( c \) between \( a \) and \( b \) at which \( f'(c) = 0 \).

**Solution:** By the Mean Value Theorem, there exists a \( c \) between \( a \) and \( b \) so that \( f'(c) = \frac{f(a) - f(b)}{a - b} = \frac{0 - 0}{a - b} = 0 \).

3. (2 points) Use 2 steps in Newton’s Method to estimate \( \sqrt{100} \), starting with \( x_1 = 2 \) (ie, find \( x_3 \)).

**Solution:** Let \( f(x) = x^7 - 100 \). Then approximating \( \sqrt[7]{100} \) is the same as approximating the root of \( f(x) \). We have \( f'(x) = 7x^6 \). Starting with \( x_1 = 2 \), we get:

\[
x_2 = 2 - \frac{2^7 - 100}{7 \cdot 2^6} = 1.9375
\]

\[
x_3 = 1.9375 - \frac{(1.9375)^7 - 100}{7 \cdot (1.9375)^6} = 1.930768
\]