1. Find a basis for the orthogonal complement of $U = \text{span}\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \}$. Hint: create a matrix $A$ with row $A = U$, then $\text{nul } A = U^\perp$.

2. Let $A$ be a $m$ by $n$ matrix whose columns are mutually orthogonal (i.e. each column is orthogonal to every other column). What is $A^tA$?

3. Compute the orthogonal projection of the vector
\[
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}
\]
onto the subspace $U$ with basis
\[
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}
\]
via the $A(A^tA)^{-1}A^t$ method.

4. Given the basis
\[
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}
\]
of the subspace $U$, using the Gram-Schmidt process, find an orthonormal basis for the space.

5. Using the above basis, what is the orthogonal projection of
\[
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]
onto $U$? Don’t use the $A(A^tA)^{-1}A^t$ method.

6. In the case of an orthonormal basis, by your computation in question 2, $A^tA = I_n$. In this case, the projection matrix $A(A^tA)^{-1}A^t$ is just $AA^t$. Compute the same projection above using the orthonormal basis along with the $A(A^tA)^{-1}A^t$ method. Compare your two answers.