1. Find the area of the quadrilateral $OABC$ on the figure below, coordinates given in brackets. [See pp. 160—163 of the book.]

2. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 7 & 1 \end{bmatrix}$$

(a) Calculate the nullspace of the matrix $A$.
(b) Let $B = A^T$. Find the rank of $B$.
(c) Find a basis for the column space of $B$.

3. Let

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ 4 & 2 & 3 \end{bmatrix}$$

(a) Find the reduced row echelon form of $A$.
(b) Do the rows of $A$ span $\mathbb{R}^3$? Explain your answer.
(c) Do the columns of $A$ span $\mathbb{R}^3$? Explain your answer.
(d) Your friend Bob claims that there exist bases $S = \{v_1, v_2, v_3\}$ and $T = \{w_1, w_2, w_3\}$ of $\mathbb{R}^3$ such that $[x]_S = A[x]_T$ for all $x$ in $\mathbb{R}^3$. Explain why this cannot possibly be true.

4. Let $A$ be an $n \times n$ matrix with integer entries.

(a) If $\text{det}(A) = 1$, show that $A^{-1}$ has integer entries.
(b) Suppose $A^{-1}$ has integer entries. What are the possibilities for $\det(A)$? Explain.

5. Find out whether the matrices

$$
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}, \begin{bmatrix}
4 & 1 \\
2 & 3
\end{bmatrix}, \begin{bmatrix}
3 & 4 \\
1 & 2
\end{bmatrix}, \begin{bmatrix}
2 & 3 \\
4 & 1
\end{bmatrix}
$$

form a basis in the space of all $2 \times 2$ matrices.

6. Find all vectors in $\mathbb{R}^3$ of length $\leq 2$ with integer entries. Which of them are orthogonal to the vector $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$?

7. The population of sapsuckers in Sapsucker Woods is described by the following model. Let $c_k$ denote the number of chicks in year $k$, let $j_k$ denote the number of juveniles in year $k$, and let $a_k$ denote the number of adults in year $k$. Then

$$
\begin{bmatrix}
c_{k+1} \\
j_{k+1} \\
a_{k+1}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0.2 \\
0.25 & 0.875 & 0 \\
0 & 0.5 & 0.8
\end{bmatrix}
\begin{bmatrix}
c_k \\
j_k \\
a_k
\end{bmatrix}
$$

Let $A$ be the matrix

$$
A =
\begin{bmatrix}
0 & 0 & 0.2 \\
0.25 & 0.875 & 0 \\
0 & 0.5 & 0.8
\end{bmatrix}
$$

(a) A vector $v$ in $\mathbb{R}^3$ is called a steady-state vector of $A$ if $Av = v$. Explain what this means in terms of the model.

(b) Find all steady-state vectors for $A$.

(c) After heavy logging in Sapsucker woods, biologists find that the model is no longer accurate. Instead, a more suitable model is

$$
\begin{bmatrix}
c_{k+1} \\
j_{k+1} \\
a_{k+1}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0.2 \\
0.25 & 0 & 0 \\
0 & 0.5 & 0
\end{bmatrix}
\begin{bmatrix}
c_k \\
j_k \\
a_k
\end{bmatrix}
$$

Under this new model, what do you think will happen to the population of sapsuckers in the long term? Explain your answer.

8. Let

$$
A =
\begin{bmatrix}
3 & 5 & 7 & 3 & 2 \\
2 & 1 & 0 & 2 & 0 \\
1 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 \\
3 & 2 & 4 & 5 & 2
\end{bmatrix}
$$

(a) Calculate $\det(A)$.

(b) Is $A$ invertible? Explain your answer.

(c) Calculate $\det(AA^T)$. 