Math 712

PROBABILISTIC METHODS IN ANALYSIS

Instructor E. B. Dynkin

Classes on Tuesday and Thursday, 10:10-11:25

Interactions between the theory of stochastic processes and the theory of partial differential equations are beneficial for both probability theory and analysis. At the beginning, mostly analytic results were used by probabilists. More recently analysts took inspiration from the probabilistic approach.

The main subject of the course is connections between linear and semilinear differential equations and the corresponding Markov processes called diffusions and superdiffusions. An emphasis will be on presenting the main ideas while avoiding technicalities. A general mathematical culture and an interest in probability or analysis (or both) are assumed rather than any specific backgrounds in stochastic processes or PDEs.

1. Introduction. Basic facts on Markov processes and martingales.
3. Construction of a diffusion with the generator $L$ by using the fundamental solution of a parabolic equation and by solving Ito’s stochastic differential equation.
4. Description of all positive solutions of the equation $Lu = 0$ in an arbitrary domain (Martin boundary theory).
6. Hitting probabilities for superdiffusions and removable singularities for solutions of the equation $Lu = \psi(u)$.
7. Classification and probabilistic expressions for positive solutions of the equation $Lu = \psi(u)$ in a smooth domain. Theory of the boundary trace.