HOMEWORK 1: Addendum

1. If \( w = \cos(x - y), x = 2r \sin s \) and \( y = r^2 \cos s \), use the chain rule to compute 
\[
\frac{\partial w}{\partial r} \quad \text{and} \quad \frac{\partial w}{\partial s}.
\]

2. Consider a metallic plate. Suppose that the temperature at the point \((x, y)\) is given by
\[
T(x, y) = 40e^{-x^2 - 2y^2}.
\]
(a) Find the rate of change of the temperature at \(P(2, 1)\) in the direction pointing towards \((0, 2)\).
(b) In what direction does the temperature increase the fastest at \(P\).
(c) Find the maximum rate of increase of \(T\) at \(P\).

3. Determine whether or not \(\mathbf{F} = (x + y^2)\mathbf{i} + (2xy + y^2)\mathbf{j}\) is conservative. If it is, find \(f\) such that \(\mathbf{F} = \nabla f\).

4. Verify Stokes' Theorem for the field \(\mathbf{F}(x, y, z) = z^2 \mathbf{i} + y^2 \mathbf{j} + xy \mathbf{k}\) on the triangle with vertices \((1, 0, 0), (0, 1, 0)\) and \((0, 0, 2)\).

5. Verify the Divergence Theorem for \(\mathbf{F}(x, y, z) = z^2 \mathbf{i} + y^2 \mathbf{j} + xy \mathbf{k}\) on the cube bounded by the planes \(x = 0, x = 1, y = 0, y = 1, z = 0\) and \(z = 1\).