MATH 4530 – Topology. HW 9

Please declare any collaborations with classmates; if you find solutions in books or online, acknowledge your sources in either case, write your answers in your own words. Please attempt all questions and justify your answers.

Write the proofs in complete sentences.

(1) Let $p : E \to B$ be a covering map. Let $f, g$ are composable paths in $B$, i.e. $f(1) = g(0)$. If $\tilde{f}, \tilde{g}$ are composable paths lifted from $f, g$, then show that $\tilde{f} \ast \tilde{g}$ is a lifting of $f \ast g$.

(2) Show that the fundamental group of a torus $S^1 \times S^1$ is isomorphic to $\mathbb{Z} \times \mathbb{Z}$ as groups (additive on $\mathbb{Z} \times \mathbb{Z}$). (Hint: generalized the proof of $\pi_1(S^1, b_0) \cong \mathbb{Z}$.)

(3) A group $G$ acts on a set $X$ from right if there is an action map $G \times X \to X, (g, x) \mapsto xg$ which satisfies $x = x1_G$ and $x(gh) = (xg)h$. Show that there is a natural right action of $\pi_1(B, b_0)$ on $p^{-1}(b_0)$ if $p : E \to B$ is a covering map. (Hint: use $\phi_e$ in Section 10.2 [L]).

(4) Let $B$ be a simply-connected space. Then any covering map $p : E \to B$ with $E$ path-connected, is a homeomorphism.

(5) Show that the map $p : S^1 \to S^1, z \mapsto z^n$ induces $p_* : \pi_1(S^1, b) \to \pi_1(S^1, b), [f] \mapsto [f]^n$. In other words, through the isomorphism in Section 10.4 [L], $\mathbb{Z} \to \mathbb{Z}, m \mapsto nm$.

References