MATH 4530 – Topology. HW 8 solutions

Please declare any collaborations with classmates; if you find solutions in books or online, acknowledge your sources in either case, write your answers in your own words.

Please attempt all questions and justify your answers.

Write the proofs in complete sentences.

1) Let \( f, g : I \to X \) be paths from \( x_0 \) to \( x_1 \) and let \( k : X \to Y \) be a continuous map.

1) Show that if \( F : f \Rightarrow p\ g \) is a path homotopy, then \( k \circ F : k \circ f \Rightarrow p\ k \circ g \) is a path homotopy.
2) Show that if \( f(1) = g(0) \), then \( k \circ (f \ast g) = (k \circ f) \ast (k \circ g) \).

2) Let \( Y \) be a discrete space. Show that if \( p : X \times Y \to X \) is the projection to the first component, then \( p \) is a covering map.

3) Let \( p : E \to B \) be a covering map.
   a) Show that if \( B \) is Hausdorff, then \( E \) is Hausdorff.
   b) Show that if \( B \) is compact and \( p^{-1}(b) \) is finite for each \( b \in B \), then \( E \) is compact.

4) Let \( S^1 := U(1) := \{ e^{i\theta} \} \subset \mathbb{C} \). Show that \( p_u : S^1 \to S^1, z \mapsto z^u \) is a covering map.

5) Let \( h, k : X \to Y \) be continuous maps and suppose \( h(x_0) = y_0 \) and \( k(x_0) = y_0 \). Show that if there is a homotopy \( F : X \times [0,1] \to Y \) from \( h \) to \( k \) such that \( F(x_0,t) = y_0 \) for all \( t \in [0,1] \), then \( h_s = k_s : \pi_1(X,x_0) \to \pi_1(Y,y_0) \).

References