You may use one sheet of your own notes. Books, calculators and other electronic devices are not permitted. Show all your work.

1. Calculate \( \int \int_A (x + y)^2 \, dx \, dy \) where \( A \) is the annulus \( 1 \leq x^2 + y^2 \leq 4 \).

2. Find the equations of the tangent planes to the paraboloid \( z = x^2 + y^2 \) that pass through both the points \((1,0,-1)\) and \((-1,0,-1)\). Hint: There are two planes which satisfy these conditions.

3. Find the dimensions of the rectangular box of maximum volume that can be cut out of a square \( 6 \times 6 \) sheet of paper in the way shown in the figure below. The box consists of a top, a bottom, and four sides. The dotted lines show where the paper is to be folded.

4. Set up a definite integral that computes the arclength of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \). Do not evaluate the integral.

5. (a) Compute the volume of the region that lies above the \( xy \)-plane and below the helicoid surface \((x, y, z) = (r \cos \theta, r \sin \theta, \theta)\), for \( 0 \leq r \leq 1 \) and \( 0 \leq \theta \leq 2\pi \).
(b) Compute the volume of the region inside the “seashell” defined by the equation \( \rho = \theta \) in spherical coordinates, for \( 0 \leq \theta \leq 2\pi \). Here are some views of what this surface looks like:
6. (a) Show that the line integral \( \int_C \frac{1}{y} \, dx + \left( \frac{1}{z} - \frac{x}{y^2} \right) \, dy - \frac{y+1}{z^2} \, dz \) is path-independent by finding a potential function.

(b) Evaluate the line integral when \( C \) is a path in the first octant from \((1, 1, 1)\) to \((2, 2, 2)\).

7. Let \( R \) be the region in \( \mathbb{R}^2 \) bounded by the graphs of the functions \( f(x) = \sin(x) - 1 \), \( g(x) = \sin(x) + 1 \), the \( y \)-axis, and the vertical line \( x = 2\pi \). Let \( C \) be the boundary curve of \( R \), oriented counterclockwise. Compute the line integral \( \int_C \left( ye^{xy} - xy \right) \, dx + \left( xe^{xy} + y^2 \right) \, dy \).

8. Find the flux \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) of the vector field \( \mathbf{F}(x, y, z) = (y, x, 1) \) through the surface of the paraboloid \( z = 4 - x^2 - y^2 \), \( z \geq 0 \), with the upward orientation.

9. Compute the flux of the vector field \( \mathbf{F} = \frac{1}{(x^2 + y^2 + z^2)^{3/2}}(x, y, z) \) through the surface \( z = e^{1-x^2-y^2} \), \( z \geq 1 \), with the upward orientation. Hint: Use Gauss’s theorem, but be careful because the surface is not closed.

10. Compute \( \iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S} \) where \( \mathbf{F} = (xz - yz^2, xz^2 + \sin(yz), xyz) \) and \( S \) is the part of the paraboloid \( z = x^2 + y^2 \) between the planes \( z = 1 \) and \( z = 2 \), oriented by the upward normal. Hint: This can be computed without doing any complicated integrations.