**Math 424: Homework due on 2/25**

1) From the Textbook: Chapter 2: 4, 5.

2) The goal of this problem is to find the Fourier transform of the function \( f \) defined by: \( f(x) = e^{-x^2} \). Recall that \( \int_{-\infty}^{\infty} e^{-x^2} \, dx = \frac{1}{\sqrt{2}} \).

   (a) Find \( C = \hat{f}(0) \).

   (b) Show that \( \hat{f}(\gamma) \) is a differentiable function of \( \gamma \) and find \( \frac{d\hat{f}}{d\gamma} \) for each \( \gamma \in \mathbb{R} \).

   (c) Use part b, to show that \( \hat{f}(\gamma) \) is a solution to the first order differential equation \( Y'(\gamma) + \frac{1}{2} \gamma Y(\gamma) = 0 \).

   (d) Find the general solution of the previous equation.

   (e) Use part a, d, to find \( \hat{f}(\gamma) \).

3) Let \( f \in L^1(\mathbb{R}) \), \( a > 0 \), \( b, c \in \mathbb{R} \) be given. Define the following functions \( f_1(x) = a^{1/2} f(ax) \), \( f_2(x) = f(x-b) \), and \( f_3(x) = e^{2\pi i cx} f(x) \).

   (a) Show that \( f_1, f_2 \) and \( f_3 \) are all in \( L^1(\mathbb{R}) \) and find the \( L^1 \)-norm of each of them.

   (b) Find the Fourier transform of \( f_1, f_2 \) and \( f_3 \) in terms of the Fourier transform of \( f \).

   (c) Applications. Find the Fourier transforms of \( f, g \) and \( h \) where \( f(x) = e^{-2x^2} \), \( g(x) = e^{\pi ix} \chi_{[0,1]}(x) \), and \( h(x) = \chi_{[3,4]}(x) \).