1. (2 pts) Estimate $e^{.01}$ and $e^{-0.01}$ by a linear approximation at 0.

Solution: Let $f(x) = e^x$. Then $f(0) = 1$ and $f'(0) = 1$. So,

$$L(x) = f(0) + f'(0)(x - 0) = 1 + x$$

$e^{0.01} = f(.01) \approx L(.01) = 1 + .01 = 1.01$

$e^{-0.01} = f(-.01) \approx L(-.01) = 1 - .01 = .99$

2. (3 points) A window has the shape of a square surmounted by a semicircle. The base of the window is measured as having width 60 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum error possible in computing the area of the window.

Solution: Let $x$ be a side of the square. Note that the total area of the window is

$$A = A_{\text{square}} + A_{\text{semicircle}} = x^2 + \frac{1}{2} \pi \left( \frac{x}{2} \right)^2 = \left( 1 + \frac{\pi}{8} \right) x^2.$$  

Using differentials, we have

$$dA = \left( 1 + \frac{\pi}{8} \right) (2x) dx.$$  

But $x = 60$ cm and $dx = .1$ cm, therefore the maximum error in computing the area is

$$dA = \left( 1 + \frac{\pi}{8} \right) (120)(.1) = 3 + \frac{3}{2} \pi \text{ cm}^2.$$

3. (5 points) A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

Solution: See last example in the text, section 4.1 for a solution to this problem. An alternate solution, using that same setup is: $\tan \theta = \frac{x}{20}$. Then $\theta = \arctan\left( \frac{x}{20} \right)$ and so

$$\frac{d\theta}{dt} = \frac{d}{dt} \arctan\left( \frac{x}{20} \right) = \frac{1}{1 + \left( \frac{x}{20} \right)^2} \cdot \frac{1}{20} \cdot \frac{dx}{dt} = \frac{20}{400 + x^2} \cdot \frac{dx}{dt}.$$

For $x = 15$ we then get

$$\frac{d\theta}{dt} = \frac{20}{625} \cdot 4 = \frac{16}{125} \text{ rad/s}.$$