HOMEWORK SOLUTIONS MATH 432 ASSIGNMENT 12

Exercise 5.21

(i) Note that $x^p - y^p = (x-y)^p$.

(ii) By (i) it is bijective and using a similar formula you can see it is a homomorphism. To see it fixes F_p, use Theorem 1.64.

(iii) Because F is surjective.

Exercise 5.22

(i) Trivial.

(ii) $F_p(\alpha) = F_p[x]/(p(x))$, and by counting their numbers of elements you could see $p(x)$ has to be of degree n.

(iii) The hint tells you everything. You will need the property that σ fixes F_p to prove that $\sigma(\alpha)$ is a root of $p(x)$.

(iv) Try to argue that $F^i : a \mapsto a^{p^i}$, $(i = 0, 1, ..., n-1)$ are pairwise different maps in $Gal(F_q/F_p)$.

Exercise 5.23

(ii) to (i), if $\sqrt{b^2 - 4ac}$ is not rational, then $f(x)$ does not have roots in $\mathbb{Q}$, therefore irreducible in $\mathbb{Q}[x]$ because it is of degree 2.

(i) to (ii), if $\sqrt{b^2 - 4ac}$ is rational, then easy to see $f(x)$ has two rational roots and reducible.

(iii) to (ii), if $\sqrt{b^2 - 4ac}$ is rational, then $\mathbb{Q}(\sqrt{b^2 - 4ac}) = \mathbb{Q}$ and the Galois group of that is trivial.

(ii) to (iii), by Theorem 5.26.