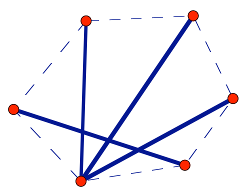
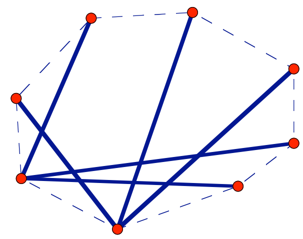
1. Let $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_n)$ be configuration of points in the real line $\mathbb{E}^1$, and let G be the (cyclic) bar graph with bars between i and $i+1$, for $i = 1, \dots, n \pmod{n}$, such that $\mathbf{p}_i \neq \mathbf{p}_{i+1} \pmod{n}$. Calculate a non-zero equilibrium stress for $G(\mathbf{p})$.

2. When does a stress that you calculated for Problem 1 have a stress matrix that is positive semi-definite of rank $n-2$?

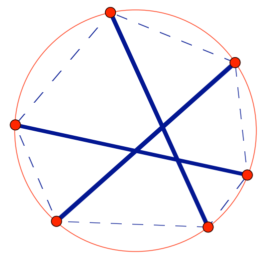
3. Prove that the following tensegrities are globally rigid in all dimensions.



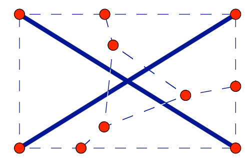
Grünbaum polygon



Roth polygon



The points lie on a circle, but are not constrained to lie on that circle



4. (a) In $\mathbb{E}^d$ consider a configuration $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_{d+2})$, where no $d+1$ lie in a $(d-1)$-dimensional affine subspace. Use the affine dependency of these points to show that the points of $\mathbf{p}$ can be partitioned into two sets, each of which form simplices σ^i and σ^j with $i+1$ and $j+1$ points respectively such that $i+j=d$, and $\sigma^i \cap \sigma^j$ is a point in the relative interior of both simplices.

   (b) Show how to place cables and struts between each pair of vertices of $\mathbf{p}$ in Part (a) such that the resulting tensegrity is globally rigid in all dimensions.

5. (a) Let $\max\{\mathbf{p}_1, \mathbf{p}_2\} < \min\{\mathbf{p}_3, \mathbf{p}_4\}$ be 4 distinct points in $\mathbb{E}^1 \subset \mathbb{E}^2$, and let G be the cyclic graph where each vertex of $\{1, 2\}$ is connected to all the vertices of $\{3, 4\}$. If $G(\mathbf{q})$ is another tensegrity with the same bar lengths as $G(\mathbf{p})$, but where the vertices of $\mathbf{q}$ do not lie in a line, show that $G(\mathbf{q})$ is embedded.

   (b) Let $\mathbf{p}_1 < \mathbf{p}_2 < \mathbf{p}_3 \cdots < \mathbf{p}_6$ be six distinct points in $\mathbb{E}^1 \subset \mathbb{E}^2$, and let G be the complete bipartite graph where each vertex of $\{1, 2, 3\}$ is connected to all the vertices of $\{4, 5, 6\}$. Show that $G(\mathbf{p})$ is globally rigid in $\mathbb{E}^2$. (It turns out that $G(\mathbf{p})$ is not even rigid in $\mathbb{E}^3$.)