# Homework for 651

Due Tuesday, March 29, 2007 (Note the later date)

1. Problem 2 on page 155 in Hatcher.

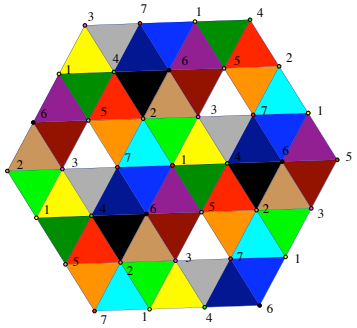
2. Problem 5 on page 155 in Hatcher.

3. Problem 14 on page 156 in Hatcher.

4. Suppose $K$ is a simplicial complex whose underlying space is homeomorphic to a closed surface of Euler characteristic $\chi$, with $v$ vertices, $e$ edges, and $f$ faces, which are triangles. Show that $2e = 3f$, $f = 2(v - \chi)$, $e = 3(v - \chi)$, and $e \leq v(v-1)/2$. Deduce that $6(v - \chi) \leq v^2 - v$. For the torus $S^1 \times S^1$ conclude that $v \geq 7$, $f \geq 14$, and $e \geq 21$.

   Consider the lattice $L \subset \mathbb{C}$ determined by all vectors of the form $m + n\eta$, where $m$ and $n$ are integers, and $\eta = \frac{1}{2}(1 + i\sqrt{3})$, and let $L' \subset L$ be the sublattice generated by $1 + 2\eta$ and $\eta^{-1}(1 + 2\eta) = \eta^{-1} + 2$. Show that $\mathbb{C}/L'$ is a covering space of $\mathbb{C}/L$, where both spaces are homeomorphic to a torus, and calculate the index of the covering map.

   Place edges between every pair of vertices in $L$ that are a unit distance apart. Show that this describes a triangulation of the torus $\mathbb{C}/L'$ with the minimal number of vertices (edges and triangles). See the figure below, where each triangle has a separate color.



   (This is part of problem 4 for Section 2.2 in the extra problems on-line on Hatcher's web page for his book.)