Homework for 651

Problems due April 17, 2007

1. Problem 19, page 157, in Hatcher.


4. One way to modify a presentation \(< g_1, \ldots, g_m | r_1, \ldots, r_n >\) to another presentation for the same group is to replace \(r_i\) with \(r_ir_j, r_ir_i^{-1}, r_jr_i,\) or \(r_j^{-1}r_i\) for some \(j \neq i.\) Show that the 2-complexes \(X_G\) associated to these different presentations are homotopy equivalent. [Hint: Attach the 2-cell \(e^2_i\) last and deform its attaching map so as to change \(r_i\) to one of the new relations, then apply Proposition 0.18.]

5. Let \(g : S^n \times I \to S^n \times I\) be continuous and \(n > 0.\) Let \(X_k = \{(x,k) \mid x \in S^n\}.\) For each fixed \(k,\) let \(g(X_k) \subset X_k.\) Further let \(f_1 : S^1 \to S^1\) be the map \(f_1(x) = 2x.\) Define \(f_i\) inductively as \(f_{i+1} = Sf_i\) where \(Sf_i\) is the suspension of the map \(f_i\) (see Hatcher page 9.) Let \(g((x,0)) = f_n.\) Suppose \(g((x,1)) = (x,1),\) does such a map \(g\) exist? What if \(g|_{X_1}\) is an odd map? What if \(g|_{X_1}\) is an even map and \(n\) is odd? What if \(g|_{X_1}\) is an even map and \(n\) is even? In each case, if a map \(g\) exists then give an example, if not prove that it does not exist.