Name:

Directions:

Complete all six questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use any notes, calculators, or other electronic devices.

You have 75 minutes.

Problem 1: ______ / 10
Problem 2: ______ / 10
Problem 3: ______ / 10
Problem 4: ______ / 10
Problem 5: ______ / 10
Problem 6: ______ / 10
Total: ______ / 60
1. Give an ordinary generating function for the sequence $a_n$ defined by $a_0 = 1$, $a_1 = 3$, and $a_n = a_{n-1} + 2a_{n-2}$ for all $n \geq 2$. 
2. For any positive integer $n$, show that there is some value of $c$ (which can depend on $n$) such that for all $k \geq c$, $p_k(n + k) = p_c(n + c)$. Also find the minimum such value of $c$. 
3. How many positive integers are there that are factors of at least one of \(2^43^7, 3^55^9, \) and \(2^85^5\)?
4. Give a formula for the Stirling number of the second kind $S(n, 2)$. 
5. Give an exponential generating function for the sequence \( a_n \) defined by \( a_0 = a_1 = 1 \) and \( a_{n+1} = a_n + n(n - 1)a_{n-1} \) for all \( n \geq 1 \).
6. Let $P$ be the set of partitions of 150 such that for all $k \geq 1$, if there is a part of size $k+1$, then there is at least one part of size $k$. Show that the number of partitions in $P$ for which the largest part is even is equal to the number of partitions in $P$ for which the largest part is odd.