Notes and books are allowed; any collaboration is not allowed.

1. (10) Find the image of the unit square \([0, 1] \times [0, 1]\) under the action of \(e^A\), \(A = \pi \left( \begin{array}{cc} -2 & 2 \\ -1 & 0 \end{array} \right)\)

2. (10) Plot the orbits of a system \(\dot{x} = Ax\) for

\[
A = \left( \begin{array}{ccc} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 2 & 2 & 0 \end{array} \right)
\]

3. (6) Solve the Cauchy problem for the wave equation on the circle 
\(S^1 = \mathbb{R}/2\pi \mathbb{Z}\): \(u_{t t} = u_{x x}, \ u|_{t=0} = 0, \ u_t|_{t=0} = \sin^3 x\)

4. (10) Find the solutions and plot the orbits in the \(x\)-plane for the following Cauchy problem: \(\ddot{x} + Ax = 0, \ x(0) = a, \ \dot{x}(0) = b\)

\[
A = \left( \begin{array}{cc} 8 & 2 \\ 2 & 5 \end{array} \right), \ a = e_1, \ b = 0
\]

5. (14) Is an operator \(\Phi : f \mapsto g, \ g(t) = t - \frac{t^3}{3} + \int_0^t \tau f(\tau) d\tau\) contracting in \(C[-\frac{1}{2}, \ \frac{1}{2}]\)? Find all its fixed points.

**Hint:** \(\int_0^t (1 - \tau^2) e^{-\frac{\tau^2}{2}} d\tau = te^{-\frac{t^2}{2}}.\)