Main part

Solve

1. (5) $\ddot{x} + 2\dot{x} + x = 0$

2. (5) $x^{(3)} - 6\ddot{x} + 12\dot{x} - 8x = 0$

3. (5) $\dddot{x} + 2\ddot{x} + x = e^t$

   Find a periodic solution with the frequency $\omega$, when exists, and plot
   the graph of its amplitude as a function of $\omega$:

4. (10) $\ddot{x} + x = \sin \omega t$

5. (10) $\ddot{x} - 0.1\dot{x} + x = \sin \omega t$

   Find a partial solution of the following equation:

6. (10) $x^{(4)} + 4x = e^t \sin \omega t$

7. (10) $x^{(3)} - 8x = e^t \cos \omega t$

Supplementary part.

8. (10) Prove that quasipolynomials with different exponents or different
   degrees are linear independent. Hint: use induction in the sum of
   degrees, and the division-derivation method.

9. (10) Find all the eigenfunctions and eigenvalues of the operator of
   taking the first derivative on the line.

10. (10) Find all the eigenfunctions and eigenvalues of the operator of
   taking the second derivative on the circle $S^1 = \mathbb{R}/2\pi \mathbb{Z}$. 