

ASSIGNMENT 413-5V (FOR VOLUNTEERS ONLY)

To be returned October 16, 2003

1 (10). For any metric space  $M$  define its *completion*  $\tilde{M}$  in the same way as it was done when constructing real numbers out of rational ones. Define a metric on  $\tilde{M}$  whose restriction onto  $M$  coincides with the original metric on  $M$ .

2 (Score depends on the example) Give an example of the construction above different from  $\mathbb{Q}$  and  $\mathbb{R}$ .

3 (20). Prove that  $\tilde{M}$  in Problem 1 is complete.

4 (5). Define a compact set in a metric space in parallel to Def. 3.3.1.

5 (10). Prove that a nested sequence of compact sets in a metric space has a nonempty intersection.

6 (20). Suppose that a metric space has a countable dense subset. Prove that any cover of a compact set in this space has a finite subcover.

7 (15). Is it true that any closed bounded subset of a metric space is compact?