Assignment 413-5. Hints to some problems

1. a) $\{-1; 1\}$
c) $\{-3; 1\}$

2. $0, -1, 0, 1, -2, -1, 0, 1, 2, -3, -2, \ldots$

3. $A = \mathbb{R} \setminus (\frac{1}{n})$ is not open: no neighborhood of 0 is contained in A.

4. A and $A \setminus x,\ x \in A$ are both closed iff x is a isolated point of A.

Proof. Let x be nonisolated. Then it is a limit point of A, that is, a limit of sequence $(x_k \in A)$ with disjoint elements. Then $x_k \in A \setminus x\ \forall k$, and $x_k \to x \notin A \setminus x$. Hence, $A \setminus x$ is not closed.

If x is isolated in A, then it is not a limit point of A. Hence, $A \setminus x$ still contains all of the limit points of A. Now, since all limit points of $A \setminus x$ are clearly also limit points of A, $A \setminus x$ contains all of its limit points. Hence $A \setminus x$ is closed.

6. a) Hint: use the diagonal process, as in the proof of the completeness of $\mathbb{R}$.
b) $\{0\}$
c) $\{0; \frac{1}{n} | n \in \mathbb{N}\}$