

ASSIGNMENT 413-3

To be returned 09.23.03

**Main Part.**

- 1 (2). Prove uniqueness of the reciprocal element to a given element of a field.
- 2 (3). Prove existence and uniqueness of the difference for two elements of a field.
- 3 (3). Prove that the reciprocal element to a positive element of an ordered field is positive.
- 4 (2). Prove that the product of two negative elements of an ordered field is positive.
5. 2.2.4.8 a,c,d (2,3,7)
6. 2.2.4.9 b,a,c (3,5,2)
- 7 (3). Find in any field  $\sum_0^{n-1} q^k$ ,  $q \neq 1$ .
- 8 (5). Prove that for real  $q$  with  $|q| < 1$ , the sequence  $(x_n = \sum_0^{n-1} q^k)$  is a Cauchy sequence.
- 9 (5). Find rational numbers represented by the following Cauchy sequences:  
 $x_k = 0.0101 \dots 01$  ( $2k$  digits after dot; binary expansion).
- 10 (5). 2.2.4.1  
Comment. There is a misprint in the statement of one of the problems 5,6. Find it.

**Supplementary part.**

- 11 (10). Is it true that any rational number has a binary expansion that is periodic beginning with some position?
- 12 (20). Prove that  $2^{\mathbb{N}}$  and  $[0, 1]$  have the same cardinality.