1. Prove that the cardinality of the line is the same as the cardinality of the plane.

2. An open set in $\mathbb{R}$ is defined as the arbitrary union of open intervals $(a, b)$. Prove that a function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if, for all open sets $U \subset \mathbb{R}$, $f^{-1}(U)$ is open.

3. Prove that there are at most a countable number of disjoint non-empty intervals in $\mathbb{R}$. (A single point is not an interval.)

4. A set $X \subset \mathbb{R}$ is called dense if for every non-empty open set $U \subset \mathbb{R}$, $U \cap X \neq \emptyset$. Prove that the countable intersection of open dense sets is dense.

5. Among $n > 1$ integers prove that the difference of some pair of them is divisible by $n - 1$.

6. An plus sign in the plane is the translate of the set

$$([-1, 1] \times \{0\}) \cup (\{0\} \times [-1, 1]).$$

Prove that there are at most a countable number of pairwise disjoint plus signs in the plane.

7. A general plus sign in the plane is the translate of the set

$$([-t, t] \times \{0\}) \cup (\{0\} \times [-t, t]),$$

for $0 < t$ real. Prove that there are at most a countable number of pairwise disjoint general plus signs in the plane with different values of $t$ possible.

8. Prove that the cardinality of the continuous functions $f : \mathbb{R} \to \mathbb{R}$ is the same as the cardinality of $\mathbb{R}$, the real numbers.

9. An algebraic number is a real (or complex) number that is the solution to a non-zero polynomial equation with rational coefficients. Prove that there are only a countable number of algebraic numbers.
10. Three points in the plane are called \textit{generic} if their six coordinates do not satisfy any non-zero polynomial equation with integer coordinates. Prove that three generic points in the plane are distinct and not collinear.