Problem 2.2.4. Solve the differential equation \( y' = (3x^2 - 1)/(3 + 2y) \).

Solution: Separate variables so that

\[
(3 + 2y)dy = (3x^2 - 1)dx
\]

and integrate to obtain

\[
3y + y^2 = x^3 - x + C.
\]

As the above equation is quadratic in \( y \), we can solve for \( y(x) \) explicitly:

\[
y(x) = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + x^3 - x + C}.
\]

Lastly, note that this solution only holds for \( x \) such that \( 3 + 2y(x) \neq 0 \).

Problem 2.2.7. Solve the differential equation \( \frac{dy}{dx} = \frac{x \cdot e^{-x}}{y + e^y} \).

Solution: Separate variables so that

\[
(y + e^y)dy = (x - e^{-x})dx
\]

and integrate to obtain

\[
\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C.
\]
Rearrange this equation as \( y(x)^2 - x^2 + 2(e^{y(x)} - e^{-x}) = 2C \), which is a solution to the differential equation provided \( y + e^y \neq 0 \).

**Problem 2.2.12.** (a) Find the explicit solution of the initial value problem \( \frac{dr}{d\theta} = \frac{r^2}{\theta} \) provided \( r(1) = 2 \).

Solution: Separate variables so that

\[
\frac{dr}{r^2} = \frac{d\theta}{\theta}
\]

and integrate to obtain

\[
-\frac{1}{r} = \ln(|\theta|) + C \implies r(\theta) = \frac{-1}{\ln(|\theta|) + C}.
\]

Of course, this solution only holds provided \( \theta \neq 0 \). As \( r(1) = 2, C = -\frac{1}{2} \) and our explicit solution becomes \( r(\theta) = \frac{2}{1 - 2\ln(|\theta|)} \).

(b) Plot the graph of this solution. Note that the respective radii at \( \theta = 0 \) and \( \theta = e^{1/2} \) are zero and infinity.

(c) The solution exists in the interval \((0, e^{1/2})\).