

[20] 1. Calvin is mixing up a potion in the sink. He has grabbed either Potion  $X$ , or possibly Potion  $Y$ . Potion  $X$  has a concentration of Polonium of 2 micro-grams per liter. Potion  $Y$  has a concentration of Polonium of 1 micro-gram per liter. Initially, the sink has 10 liters of fluid in it, with 50 micro-grams of Polonium. Calvin adds one of the two potions to the sink at the rate of two liters per minute, mixing it well. The sink drains at the rate of one liter per minute. Once the amount of fluid in the sink has doubled, the amount of Polonium in the tank has increased to 55 micro-grams.

(a) Did Calvin use Potion  $X$  or Potion  $Y$ ? (you might need to answer this after part (b))

ans:  $X$

(b) Let  $P(t)$  denote the number of micrograms of Polonium in the sink at time  $t$ . Find  $P(t)$ .

$$\text{ans: } P(t) = \frac{A t^2 + 20 A t + 500}{10 + t} \quad \text{where } A = \begin{cases} 2 & \text{if potion } X \\ 1 & \text{if potion } Y \end{cases}$$

(c) The Federal authorities arrive at time  $t = 20$  minutes. If they detect more than 2 micro-grams of Polonium per liter in the sink, Calvin might get a vacation at Supermax. Does he get this vacation?

ans: YES!!!